

Magnetically Induced "Dry" Water Like Structure of Charged Fluid at the Core of a Magnetar

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It is shown that charged fluid, e.g., electron gas or proton matter at the core of a magnetar exhibit super-fluid (frictionless) like property if the magnetic field strength is high enough to populate only the zeroth Landau levels.

One of the oldest subject- *the effect of strong magnetic field on dense charged particle system* has got a fresh life with the observational discovery of a few magnetars. These exotic objects are assumed to be the strongly magnetized young neutron stars and also the possible sources of soft gamma repeaters (SGR) and anomalous X-ray pulsars (AXP) [1–4]. During the past few years a lot of work have been done on the effect of strong magnetic field on various physical properties of dense astrophysical matter [5–9]. An intensive studies have also been done on the structural deformation of neutron stars by strong magnetic field, emission of gravity waves from such rotating deformed objects etc [10–14].

From the observational data of SGR and AXP, the strength of surface magnetic field of magnetars are predicted to be $\geq 10^{15}$ G. Then by scalar Virial theorem it is very easy to show that the magnetic field strength at the core region may go up to 10^{18} G. This is of course strong enough to affect most of the physical processes taking place at the core region of magnetars. In particular, the physical properties like equation of states of dense hadronic matter, electromagnetic and weak processes, quark-hadron phase transition, transport properties etc. should change significantly in presence of such strong magnetic field [5–9]. Since the hadronic matter is assumed to be in β -equilibrium, in presence of a strong magnetic field, the properties of neutron sector will also change significantly. It is known that in the relativistic limit, if the cyclotron quantum of the i th charged species exceeds the rest mass of the constituent, the quantum mechanical effect of strong magnetic field on the i th charged component becomes significant. In other wards, the Landau levels of the i th charged species are populated beyond this critical field strength. This physical phenomenon is called the *Landau diamagnetism* [15]. The critical value of magnetic field strength for the i th charged component is given

by $qB^{(c)(e)} = m_i^2$, where q is the magnitude of charge carried by the particle and m_i is the rest mass of the particle (we have assumed $\hbar = c = k_B = 1$). For electron of rest mass $m_e = 0.5$ MeV, this critical strength is $B^{(c)(e)} \approx 4.4 \times 10^{14}$ G. In the case of proton, this typical value changes to $B^{(c)(e)} m_p^2 / m_e^2$, which is extremely high to achieve even at the core region of a magnetar with extremely strong magnetic field. Therefore, the quantum mechanical effect of strong magnetic field may be neglected in the case of protons. However, if the kinetic pressure of proton matter in presence of strong external magnetic field is compared with the corresponding kinetic pressure of magnetically affected electron gas, the proton pressure exceeds the electronic contribution under charge neutrality condition and in β -equilibrium (with neutron). This is obviously unphysical, since protons are much more massive than electrons. To remove this anomaly, we assume, that protons are also affected quantum mechanically by strong magnetic field. We further consider the simplest physical picture of $n - p - e$ system in β -equilibrium and also assume that $p - e$ system is charge neutral. Now it is generally believed that under such extreme physical condition at the core region of a neutron star the neutron matter exhibits super-fluidity, whereas the proton sector becomes super-conducting [16]. However, in the present article, we are not interested on the type of super-conductivity of proton matter. Since the kinetic energy of electrons are a few orders of magnitude larger than the super-conducting band gap obtained from BCS theory, they never show super-conducting behavior. Now following the relativistic theory of superfluidity and super-conductivity of Fermionic system, developed by Bailin and Love [17], we have estimated the critical strength of magnetic field at which the superconductivity of proton matter is completely destroyed. The typical value is $\sim 10^{16}$ G corresponding to the density and temperature relevant for neutron star core. Therefore, at the core region of a magnetar the possibility of proton super-conductivity may be ruled out. However, the super-fluidity of neutron matter remains unaffected.

In this article we shall try to show from the study of relativistic transport theory of dense electron gas or proton matter, that in presence of an ultra-strong magnetic field, the shear viscosity coefficient vanishes or becomes negligibly small. In fact, when only the zeroth Landau levels are occupied by the charged particles, the shear viscosity coefficient vanishes identically. In fig.(1) we have shown the curve ($B - n_B$ diagram) that separates the region where

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only zeroth Landau levels are populated (indicated by "zero" in the figure) and the region where other non-zero Landau levels are also occupied by the electrons (indicated by "non-zero" in the figure). The curve corresponding to proton matter exactly coincides with the curve for electron gas. We have further noticed that nature of the curve is insensitive with the increase in temperature as long as it is $< 40\text{MeV}$. As the temperature increases further, the curve simply goes up, i.e., at higher temperature much stronger magnetic field is needed to populate only the zeroth Landau levels. Therefore, if one crosses the curve from below, enters the zero Landau level zone both for electron and proton. This curve also indicates that in the region "zero" the charged fluid is frictionless (zero shear viscosity). However, the charged fluid, in particular the proton matter is not super-conducting in this region. The electrical conductivities of both electrons gas and proton matter remain finite in this zone.

Now to show that the shear viscosity coefficient of charged particle system vanishes in presence of extremely strong magnetic field or in other words, in the "zero" region as shown in the figure, at which only the zeroth Landau levels are occupied, we start with the conventional form of relativistic Boltzmann equation following de Groot [18]. The general form of relativistic Boltzmann equation is given by

$$[p^\mu \partial_\mu + \Gamma_{\nu\lambda}^\mu p^\nu p^\lambda \partial_{p_\mu}]f(x, p) = C[f] \quad (1)$$

where $f(x, p)$ is the distribution function for electron or proton and $C[f]$ is the collision term which contains the rates of all possible elementary processes. The second term on the left hand side is the external force field term. In the case of flat space-time geometry or in absence of an external field this term becomes zero. This particular term also does not play any direct significant role in the evaluation of viscosity coefficient.

To obtain an expression for the shear viscosity coefficient of charged fluid, e.g., electron gas or proton matter, we follow the theoretical technique described in the famous book on Relativistic Kinetic Theory by de Groot [18]. We replace collision term by the relaxation time approximation, given by

$$C[f] = -\frac{p^0}{\tau}(f(x, p) - f^0(p)) \quad (2)$$

where τ is the relaxation time, which again depends on the rates of all the relevant (strong, electromagnetic and weak) processes and $f^0(p)$ is the equilibrium distribution function (in this case it is the Fermi distribution of either electron or proton). Let us consider a fluid element moving with the hydrodynamic four velocity u^μ , then we can define a second rank tensor $\Delta^{\mu\nu} = g^{\mu\nu} - u^\mu u^\nu$ which projects the space-like component of any general four vector, say a^μ . Where $g^{\mu\nu} = \text{diag}(1 - 1 - 1 - 1)$ is the metric tensor. Hence it is possible to define the convective time derivative $D = u^\mu \partial_\mu$ and gradient operator $\nabla^\mu = \Delta^{\mu\nu} \partial_\nu$. The zeroth order dynamical equations for

electron gas or for the proton matter are then obtained from the stability condition of Boltzmann equation and the first order decomposition of $(Df_k)^{(1)}$, given by (see ref. [18])

$$(Df_k)^{(1)} = \frac{\partial f_k^0}{\partial n_k}(Dn_k)^{(1)} + \frac{\partial f_k^0}{\partial T}(DT)^{(1)} + \frac{\partial f_k^0}{\partial u^\mu}(Du^\mu)^{(1)} \quad (3)$$

where k represents either electron or proton. The stability equations are obtained from the conservation laws of charge current and energy momentum tensor Then we have for the electron gas

$$\begin{aligned} Dn &= -n \nabla_\mu u^\mu \\ Du^\mu &= \frac{1}{nh} \nabla^\mu P \\ Cv DT &= F(\mu_e, T) \nabla_\mu u^\mu \end{aligned} \quad (4)$$

Similar sets of equation can also be obtained for proton matter. To obtain the set of dynamical equations (eqn.(4)) for either electron gas or proton matter, we have assumed that $n - p - e$ system is a reactive mixture and followed the theoretical techniques as discussed in ref. [18] for such a system. In the above equations, n is the number density, P is the kinetic pressure, $h = \varepsilon + Pn^{-1}$ is the enthalpy per electron, ε is the average energy per particle, $C_v = \partial\varepsilon/\partial T$ is the specific heat per particle at constant volume for the electron gas or proton matter and $F(\mu_i, T) = P(\mu_i, T)/n(\mu_i, T)$. The function $F(\mu_i, T) = T$ in the classical case, here $i = e$ or p .

It is further assumed that the system is very close to its equilibrium configuration. Then we can linearize the transport equation by the ansatz

$$f(x, p) = f^0(p)(1 + \chi(x, p)) \quad (5)$$

where $\chi(x, p)$ is the first order deviation from equilibrium configuration. Now the standard technique which is generally followed to obtain the transport coefficients of electron gas or of proton matter is to linearize Boltzmann equation using the above ansatz and decompose partial derivative into convective time derivative and gradient operator and finally use the dynamical equations (eqn.(4)) and the relativistic version of Gibbs-Duhem equation, given by

$$\frac{1}{n} \nabla^\mu P = h \frac{\nabla^\mu T}{T} + T \nabla^\mu \left(\frac{\mu_e}{T} \right) \quad (6)$$

where μ_e is the electron chemical potential. Combining all these, we finally get

$$\chi(x, p) = \frac{\tau(1 - f^0)}{p^0 T} (QX + p^\mu p^\nu \bar{X}_{\mu\nu} - p_\nu (p^\mu u_\mu - h) X_q^\nu) \quad (7)$$

where X , $\bar{X}_{\mu\nu}$ and X_q^ν are the driving forces for volume viscosity, shear viscosity and heat conduction respectively. In the present article we are only interested on the driving force for shear viscosity, given by

$$\bar{X}_{\mu\nu} = (\nabla_\mu u_\nu - \frac{1}{3}\Delta_{\mu\nu}\nabla_\sigma u^\sigma) \quad (8)$$

The next step is to substitute the linearized form of distribution function in the expression for energy momentum tensor, given by

$$T^{\mu\nu} = \sum_{\nu=0}^{\infty} \int \frac{d^3 p}{p^0(2\pi)^3} p^\mu p^\nu f(x, p) \quad (9)$$

In this case the phase space volume element $d^3 p = dp_z eB/(2\pi^2)$ and ν is the Landau quantum number. Substituting the linearized form of $f(x, p)$, we get

$$T^{\mu\nu} = T_{\text{eq}}^{\mu\nu} + T_{\text{non}}^{\mu\nu} \quad (10)$$

where

$$T_{\text{eq}}^{\mu\nu} = \epsilon u^\mu u^\nu - P \Delta^{\mu\nu} \quad (11)$$

and

$$T_{\text{non}}^{\mu\nu} = \sum_{\nu=0}^{\infty} \int \frac{d^3 p}{p^0(2\pi)^3} p^\mu p^\nu \chi(x, p) f^0(p) \quad (12)$$

are respectively the equilibrium and non-equilibrium parts of energy momentum tensor. To obtain an expression for shear viscosity coefficient of charged fluid, we consider a special kind of flow. We assume that the magnetic field is along the positive z -direction and the flow is also in the same direction. Then we have

$$T_{\text{non}} = -\eta \frac{\partial u_z}{\partial r} \quad (13)$$

where η is the coefficient of shear viscosity. Substituting the expression for χ in the non-equilibrium part of energy momentum tensor, putting $\mu = r$ and $\nu = z$, and finally equating the coefficient of the driving force $\partial u_z/\partial r$, we get

$$\eta = \frac{1}{T} \sum_{\nu=0}^{\infty} \int \frac{d^3 p}{p^0(2\pi)^3} \tau(p_z)(p_r p_z)^2 f^0(p)(1 - f^0(p)) \quad (14)$$

Let us first consider the electron gas. In this case if the magnetic field strength is greater than the critical value $B^{(c)(e)}$, the modified form of single particle energy is given by $\varepsilon_\nu = (p_z^2 + m_e^2 + 2\nu eB)^{1/2}$ and the transverse momentum $p_r = (2\nu eB)^{1/2}$, where ν is the Landau quantum number. It is known that with the increase of magnetic field strength the maximum value ν_{\max} up to which the Landau levels are occupied by electrons, decreases and in the limiting case, when B becomes extremely high, ν_{\max} becomes zero. As indicated in fig.1, it also depends on the density of electron gas. In this extreme physical scenario the transverse part of electron momentum p_r also vanishes and as a consequence the shear viscosity coefficient of electron gas (see eqn.(14)) also becomes zero. The same is also true for the proton matter.

Therefore in the extreme physical situation, when the magnetic field strength is sufficiently high, the viscosity coefficient of charged fluid at the core region of a magnetar vanishes, which means the matter becomes frictionless, i.e., the charged fluid, including the electron gas behaves "like" super-fluid. Unlike the dense hadronic matter at the core region of a normal neutron star, where (i) neutron matter shows super-fluidity, (ii) proton matter also exhibits super-fluid "like" property, but it does not show super-conductivity and (iii) the electron gas also shows super-fluid "like" behavior. Therefore all the constituents at the core of a magnetar in this simplified picture behave like frictionless fluid without the superconducting properties of proton matter.

Because of the presence of dissipative processes at the core region of a magnetar, entropy will be produced through irreversible processes. The entropy production σ is defined as

$$\sigma = \partial_\mu s^\mu \quad (15)$$

where s^μ is the entropy current. It can be shown that, in the first Chapman-Enskog approximation, the entropy production because of shear viscosity only is given by

$$\sigma = \frac{1}{T} \bar{\Pi}^{\mu\nu} \bar{X}^{\mu\nu} \quad (16)$$

where $\bar{\Pi}^{\mu\nu}$ is the traceless part of viscous pressure tensor. Since the matter becomes frictionless (non-viscous) in presence of ultra strong magnetic field, the entropy production simply becomes zero and right hand side of eqn.(16) vanishes. Since the elementary processes taking place at the core region are not stopped by the strong magnetic field, a chemical evolution of the matter is possible in this extreme physical condition without the generation of entropy. In reality, the entropy will be produced through other irreversible processes, e.g., volume viscosity and heat conduction which are non-zero in such exotic matter. This is of course very very strange behavior of charged fluid. The charged fluid behaves like "dry" water if only the zeroth Landau levels are populated in presence of strong quantizing magnetic fields.

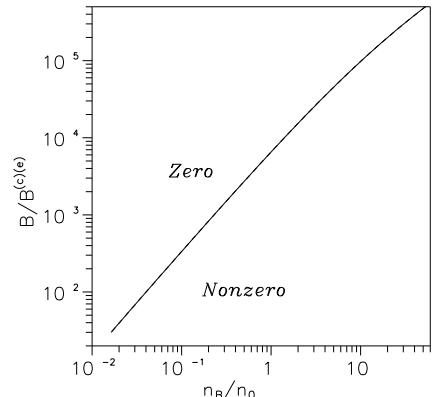


FIG. 1. The curve separating only zero Landau level occupied region and non-zero Landau level occupied region.

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